Facit till diagnosprov Ma5, kap 4 ①

1) $a_1 = 5$

$a_2 = a_1 + 2 \cdot 1^2 = 5 + 2 = 7$

$a_3 = a_2 + 2 \cdot 2^2 = 7 + 2 \cdot 4 = 15$

Svar: 5, 7, 15

2) b) 3, 7, 11, 15, ...

$d =$ 4, 4, 4, ...

Ansats: $a_n = 4n + \text{const.}$

$a_1 = 4 \cdot 1 + \text{const.} = 3$

$\text{const.} = 3 - 4 = -1$

$a_n = 4n - 1$

a) Rekursion:

$a_{n+1} = a_n + 4$

3) $a_n = d \cdot n + (a_1 - d)$ $\quad \begin{cases} a_1 = 80 \\ d = 4 \end{cases}$

$a_{40} = 4 \cdot 40 + (80 - 4) = 160 + 76 = 236$

~~$S_n = \frac{n}{2} \cdot (a_1 + a_n)$~~

$S_{40} = 20 \cdot (80 + a_{40}) = 20 \cdot (80 + 236) =$

$= 20 \cdot 316 = 6320$

4) $S_n = \frac{n}{2}(a_1 + a_n)$, men $n = ?$

Därför: $a_n = d \cdot n + (a_1 - d)$ $\quad \begin{cases} a_1 = 200 \\ d = -10 \end{cases}$

$30 = -10n + (200 + 10)$

$30 = -10n + 210$

$10n = 210 - 30 = 180$

$\underline{n = 18}$

$S_{18} = \frac{18}{2} \cdot (200 + 30) = 9 \cdot 230 = \underline{2070}$

5) $S_{10} = 200 \cdot \frac{(\frac{1}{4})^{10} - 1}{\frac{1}{4} - 1} = 200 \cdot \frac{1 - (\frac{1}{4})^{10}}{\frac{3}{4}} =$

$= \frac{800}{3} \cdot \left(1 - \frac{1}{4^{10}}\right) = \frac{800}{3} \cdot (1 - 4^{-10}) =$

$\underline{267}$

6) $a_n = a_1 \cdot k^{n-1} = x \cdot 0{,}8^9 \Rightarrow \begin{matrix} a_1 = x \\ n = \underline{10} \end{matrix}$

$S_{10} = x \cdot \frac{0{,}8^{10} - 1}{0{,}8 - 1} = 15\,000$

$x = \frac{15\,000 \cdot 0{,}2}{1 - 0{,}8^{10}} = \underline{3360{,}87}$

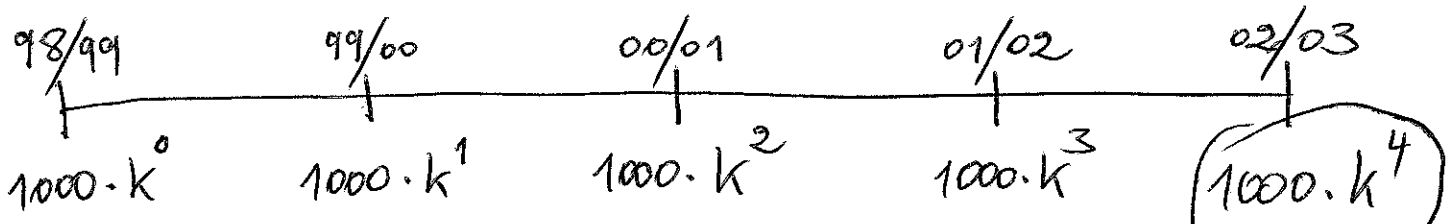
7) x = nuvärdet

$x \cdot 1{,}048^8 = 100\,000$

$x = \frac{100\,000}{1{,}048^8} = \underline{68\,724}$

8) $S_5 = 1000 \cdot \frac{1{,}055^{5} - 1}{1{,}055 - 1} = \underline{5\,581}$

Varför 5 ? $k = 1{,}055$



$a_1 + a_2 + a_3 + a_4 + a_5$

$S_5 = a_1 + \cdots + a_5$: Fem summander

$a_n = a_1 \cdot k^{n-1}$ dvs $a_5 = a_1 \cdot k^{5-1}$

9) $n = 20$ $a_1 = x$

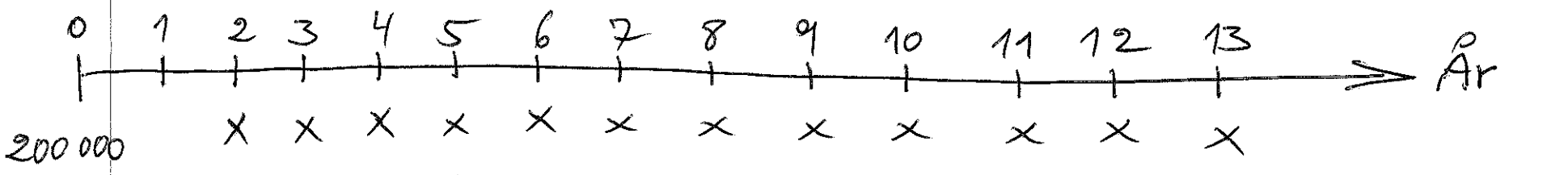
$S_{20} = x \cdot \frac{1{,}055^{20} - 1}{1{,}055 - 1} = 250\,000$

$x = \frac{250\,000 \cdot 0{,}055}{1{,}055^{20} - 1} =$

$= \underline{7170}$

10)

10)



Värdet av 12 annuiteter med ränta = Värdet av 200 000 med 13 års ränta

$$x \cdot \frac{1{,}092^{12} - 1}{1{,}092 - 1} = 200\,000 \cdot 1{,}092^{13}$$

$$x = \frac{200\,000 \cdot 1{,}092^{13} \cdot 0{,}092}{1{,}092^{12} - 1} = \underline{30\,808}$$

11) 16 % bryts ner $\Rightarrow$ 84 % kvar i kroppen.

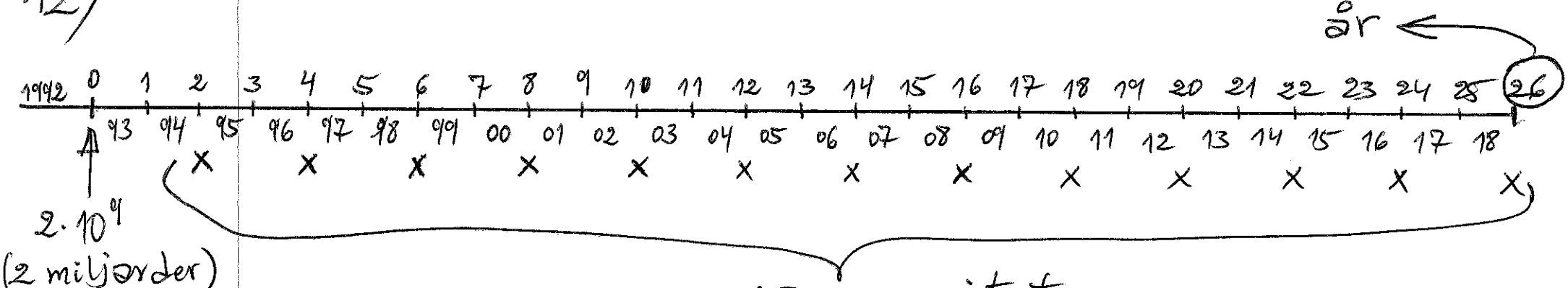
x = 1 dos

10 doser: (1:a dosen)

$$s_{10} = x \cdot 0{,}84^{9} + x \cdot 0{,}84^{8} + \cdots + x \cdot 0{,}84 + x = 15 \text{ mg}$$

$$s_{10} = x \cdot \frac{0{,}84^{10} - 1}{0{,}84 - 1} = 15 \Rightarrow x = \frac{15 \cdot (-0{,}16)}{0{,}84^{10} - 1} = \underline{2{,}9 \text{ mg}}$$

12)



årlig räntesats: 8,5 %

2-årig " : 2·8,5% = 17%

Värdet av 13 "annuiteter" med 17% räntesats =

Värdet av 2 miljarder med 26 års ränta med 8,5% räntesats:

$$x \cdot \frac{1{,}17^{13} - 1}{1{,}17 - 1} = 2 \cdot 10^9 \cdot 1{,}085^{26}$$

$$x = \frac{2 \cdot 10^9 \cdot 1{,}085^{26} \cdot 0{,}17}{1{,}17^{13} - 1} = \underline{\underline{423 \text{ miljoner}}}$$