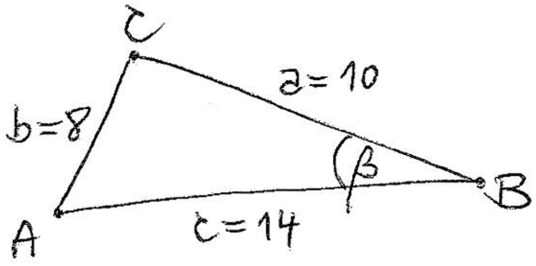
Lösningar till diagnosprov Ma4 kap 1

(1)



Cosinussatsen:

$$b^2 = a^2 + c^2 - 2ac \cdot \cos\beta$$

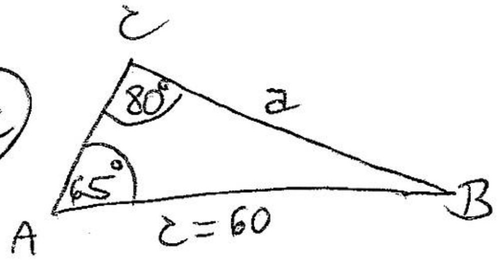
$$2ac \cdot \cos\beta = a^2 + c^2 - b^2$$

$$\cos\beta = \frac{10^2 + 14^2 - 64}{2 \cdot 10 \cdot 14}$$

$$\beta = \cos^{-1}\left(\frac{100 + 196 - 64}{280}\right)$$

$$\underline{\underline{\beta = 34^\circ}}$$

(2)



$B = 180^\circ - (65^\circ + 80^\circ)$

$B = 35^\circ$

Sinussatsen:

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

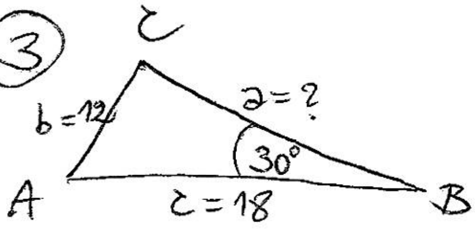
$$\frac{\sin 65^\circ}{a} = \frac{\sin 80^\circ}{60}$$

$$\frac{60 \cdot \sin 65^\circ}{\sin 80^\circ} = a$$

Areasatsen: $A = \frac{a \cdot c \cdot \sin B}{2} = \frac{60 \cdot \sin 65^\circ \cdot 60 \cdot \sin 35^\circ}{2 \cdot \sin 80^\circ}$

$$\underline{\underline{A = 950{,}14}}$$

(3)



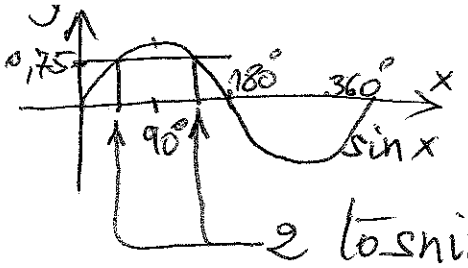
$\sin B = \sin 30^\circ = \frac{1}{2}$

Sinussatsen:

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\sin C = \frac{18 \cdot \sin 30^\circ}{12} = \frac{\cancel{18}^{\,3} \cdot 1}{\cancel{12}_{\,2} \cdot 2}$$

$$\sin C = \frac{3}{4}$$



2 lösningar: $\begin{cases} C_1 = 48{,}5904° \\ C_2 = 180° - C_1 = 131{,}4096° \end{cases}$ where $C = \sin^{-1}(0{,}75)$

$$C = \sin^{-1}(0{,}75)$$

$$\Downarrow$$

$$A_1 = 180° - 30° - C_1 = 101{,}4096°$$

$$A_2 = 180° - 30° - C_2 = 18{,}5904°$$

Sinussatsen: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$

$$\frac{\sin 101{,}4096}{a_1} = \frac{1}{24}$$

$$24 \cdot \sin 101{,}4096 = a_1$$

$$\underline{\underline{a_1 = 23{,}53}}$$

$$24 \cdot \sin 18{,}5904° = a_2$$

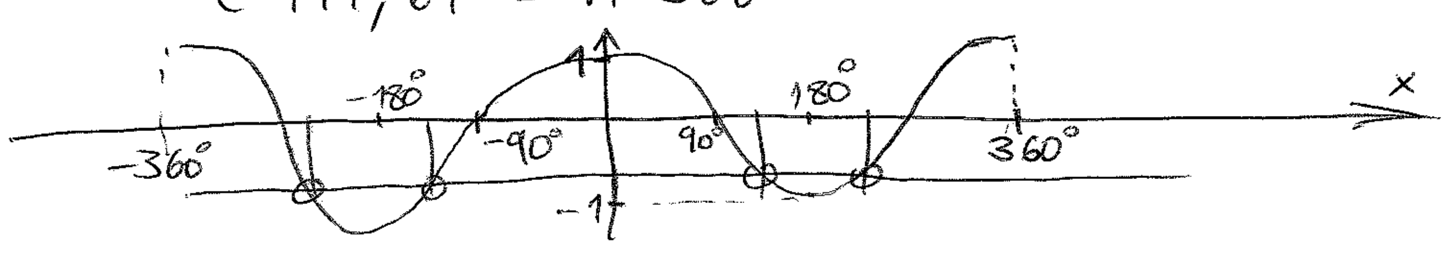
$$\underline{\underline{a_2 = 7{,}65}}$$

2 lösningar
2 svar
båda o.k.

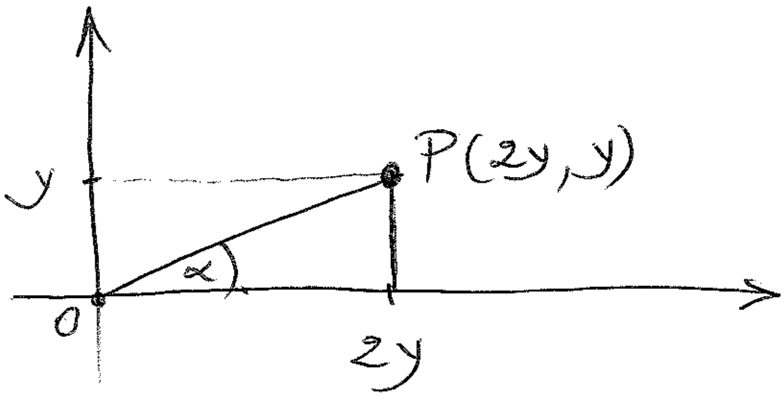
(4) $\cos v = -0{,}845$

$$v = \cos^{-1}(-0{,}845) = 147{,}67°$$

Svar: $\begin{cases} 147{,}67° + n \cdot 360° \\ -147{,}67° - n \cdot 360° \end{cases}$ (n heltal)



(5)



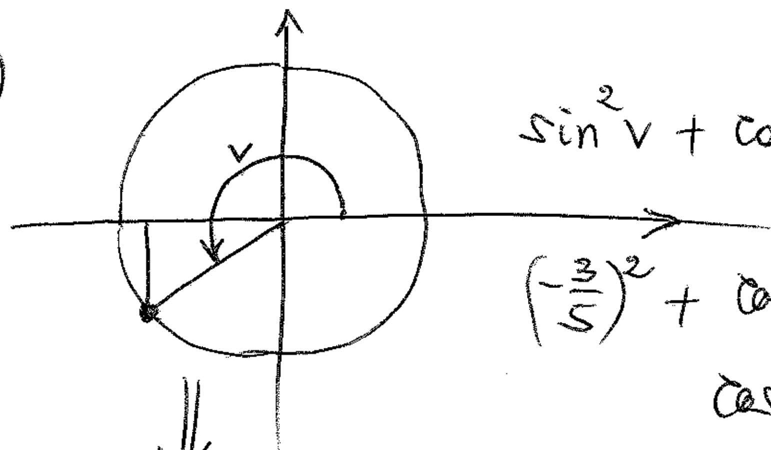
$$\tan\alpha = \frac{y}{2y}$$

$$\tan\alpha = \frac{1}{2}$$

$$\alpha = \tan^{-1}\left(\frac{1}{2}\right)$$

$$\underline{\underline{\alpha = 26{,}6^\circ}}$$

(6)



$$\sin^2 v + \cos^2 v = 1$$

$$\left(-\frac{3}{5}\right)^2 + \cos^2 v = 1$$

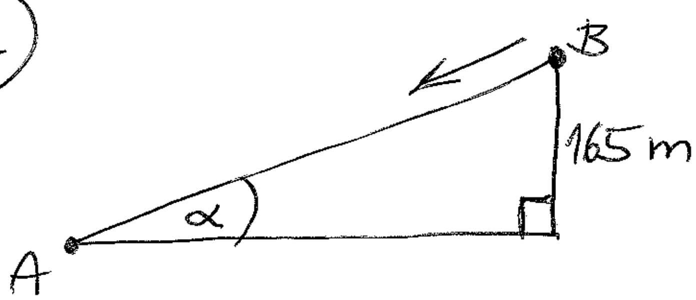
$$\cos^2 v = 1 - \frac{9}{25} = \frac{16}{25}$$

$$\cos v = \pm\frac{4}{5}$$

$\Downarrow$

Svar: $\underline{\underline{\cos v = -\frac{4}{5}}}$

(7)



AB = nedfartens längd

$$67\ m/s = \frac{AB}{10\ s}$$

$$670\ m = AB$$

$$\sin\alpha = \frac{165\ m}{670\ m} = 0{,}2463$$

$$\alpha = \sin^{-1}(0{,}2463)$$

$$\underline{\underline{\alpha = 14^\circ}}$$

(8) $\tan\left(\frac{x}{4}\right) = \frac{1}{2}$

$$\frac{x}{4} = \tan^{-1}\left(\frac{1}{2}\right) = 26{,}565^\circ$$

$$x = 4 \cdot 26{,}565^\circ = 106{,}26^\circ$$

$$\frac{x}{4} = 180^\circ$$

$$x = 4 \cdot 180^\circ$$

$$x = 720^\circ$$

Svar: 720°

$\underline{\underline{106^\circ + n \cdot}}$ (n heltal)

(9) $14{,}4 \cdot \cos x + 6 \cdot \sin x = 0$

$$6 \cdot \sin x = -14{,}4 \cdot \cos x \quad \Big| \, / \cos x$$

$$6 \cdot \frac{\sin x}{\cos x} = -14{,}4 \cdot \frac{\cos x}{\cos x}$$

$$6 \cdot \tan x = -14{,}4$$

$$\tan x = -\frac{14{,}4}{6}$$

$$x = \tan^{-1}\left(-\frac{14{,}4}{6}\right)$$

$$x = -67{,}38^\circ$$

Svar: $x = -67{,}4^\circ + n \cdot 360$

$n=1$

eller $\hookrightarrow$ $x = 292{,}6^\circ + n \cdot 360$

(10) Påstaudet: $\tan x = \dfrac{\sin 2x}{1 + \cos 2x}$

Bevis: $HL = \dfrac{2 \cdot \sin x \cdot \cos x}{1 + \cos^2 x - \sin^2 x} =$

$$= \frac{2 \cdot \sin x \cdot \cos x}{1 + \cos^2 x - (1 - \cos^2 x)} = \frac{2 \cdot \sin x \cdot \cos x}{\cancel{1} + \cos^2 x \cancel{- 1} + \cos^2 x} =$$

$$= \frac{\cancel{2} \cdot \sin x \cdot \cancel{\cos x}}{\cancel{2} \cos^{\cancel{2}} x} = \frac{\sin x}{\cos x} = \tan x = VL$$

(11) Påstående: $\dfrac{\cos x}{1+\sin x} = \dfrac{1}{\cos x} - \tan x$

Bevis: $VL = \dfrac{\cos x}{1+\sin x} = \dfrac{\cos x \cdot (1-\sin x)}{(1+\cancel{\cos}\,\sin x)\cdot(1-\sin x)} =$

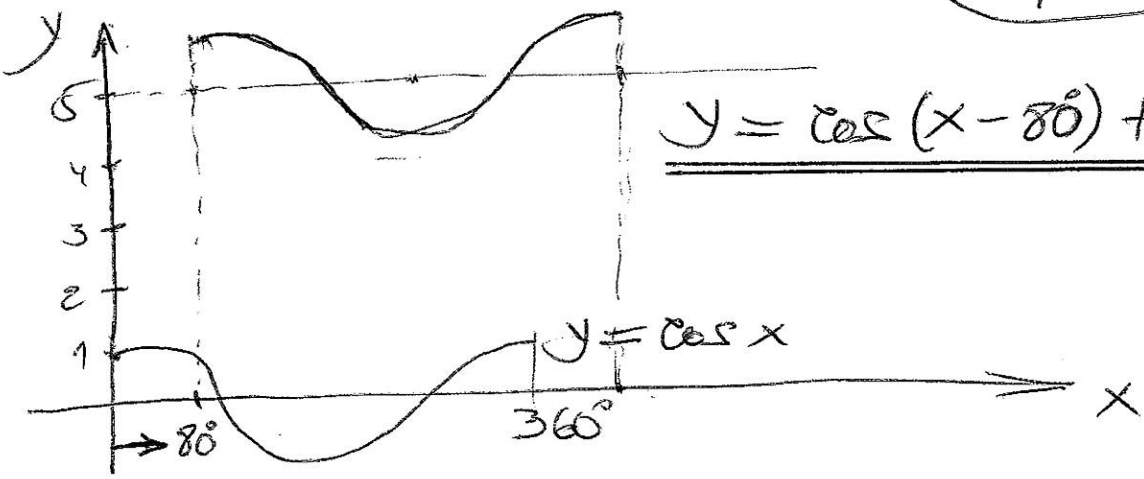
$$= \frac{\cos x \cdot (1-\sin x)}{1-\sin^2 x} = \frac{\cancel{\cos x} \cdot (1-\sin x)}{\cancel{\cos^2 x}} =$$

$$= \frac{1-\sin x}{\cos x} = \frac{1}{\cos x} - \frac{\sin x}{\cos x} =$$

$$= \frac{1}{\cos x} - \tan x = HL$$

Lösningar till diagnosprov Ma 4 kap 2

(1)

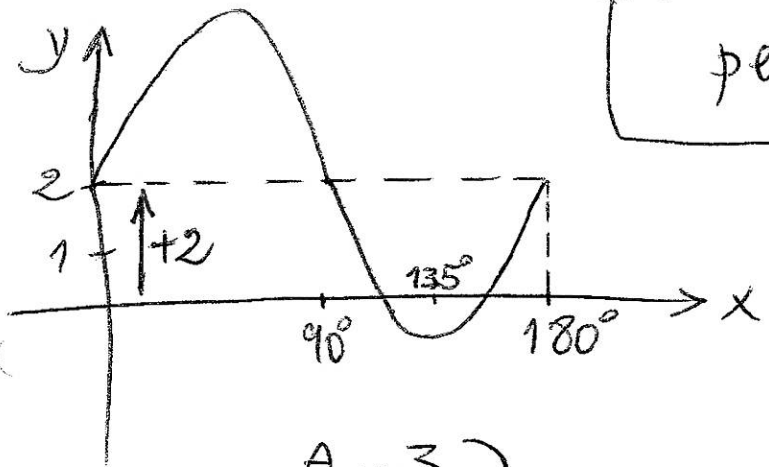


$$\underline{\underline{y = \cos(x - 80°) + 5}}$$

(2) Amplituden $A = \dfrac{y_{max} - y_{min}}{2} = \dfrac{5-(-1)}{2} = 3$

Frekvensen k = 2, därför att:

periodden = $180° = \dfrac{360°}{2}$



Förskjuten i y-led med 2 enheter

$\Downarrow$

$B = 2$

Svar: $\left.\begin{matrix} A = 3 \\ B = 2 \\ k = 2 \end{matrix}\right\} : \underline{\underline{y = 3 \cdot \sin 2x + 2}}$

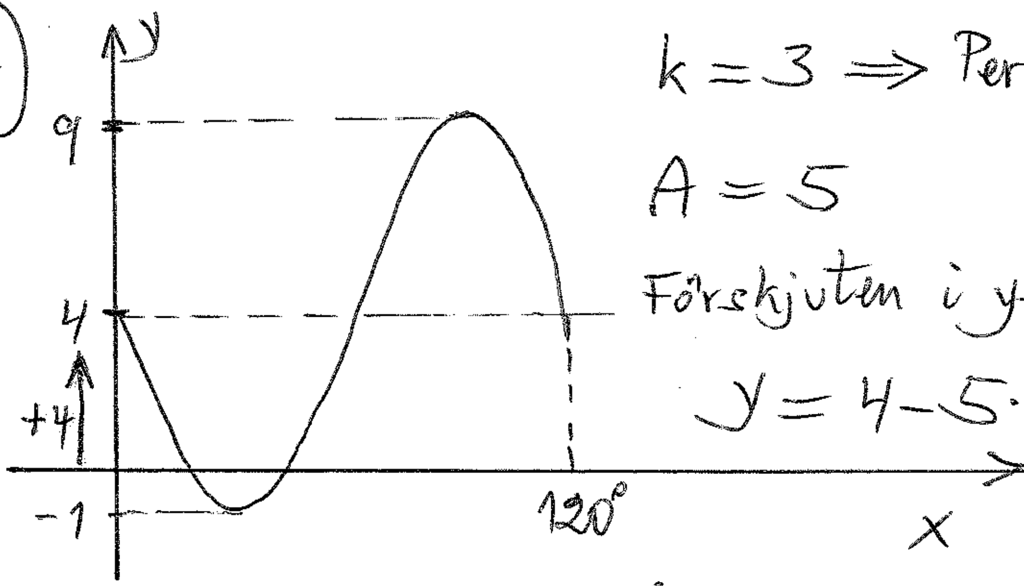
(Beteckningar: se ovan)

(3)

$$\left.\begin{matrix} A = \dfrac{5-(-25)}{2} = 15 \\ k = \dfrac{360°}{40°} = 9 \\ B = -10 \end{matrix}\right\} : \underline{\underline{y = 15 \cdot \cos 9x - 10}}$$

40° → kurvans period

(4)



$k = 3 \Rightarrow \text{Period} = \frac{360^\circ}{3} = 120^\circ$

$A = 5$

Förskjuten i y-led med 4 enheter:

$y = 4 - 5 \cdot \sin 3x$

Svar:

Period $= 120^\circ$

Största värde $= 4 + 5 = 9$

Minsta värde $= 4 - 5 = -1$

(5) $\text{Period} = \frac{360^\circ}{2} = 180^\circ$

$A = 10$

Förskjutning i y-led $= 10$

——— " ——— " x-led $= 10^\circ$

Max-värde $= 10 + 10 = 20$

Min-värde $= 10 - 10 = 0$

OBS!

$\ldots \sin(2x - 20^\circ)$

$\frac{20^\circ}{2} = 10^\circ$

$20^\circ = 10^\circ \cdot 2$

av samma anledning

(6) $\text{Period} = 720^\circ \Rightarrow k = \frac{360^\circ}{720^\circ} = \frac{1}{2}$

Förskjutning i x-led $= 120^\circ$

$120^\circ \cdot \frac{1}{2} = 60^\circ$

$A = 1$

Ingen förskjutning i y-led:

$y = \sin\left(\frac{x}{2} - 60^\circ\right)$

Pröva även:

$\sin\left(\frac{120^\circ}{2} - 60^\circ\right) =$

$\sin 0^\circ = 0$

(7) $1 \text{ rad} = \frac{180^\circ}{\pi}$

$$\frac{5\pi}{36} \text{ rad} = \frac{5\pi}{36} \cdot \frac{180^\circ}{\pi} = \frac{5\cancel{\pi} \cdot \cancel{180^\circ}^{10^\circ}}{\cancel{36}_{2} \cdot \cancel{\pi}} =$$

$$= \underline{\underline{25^\circ}}$$

(8)

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

$$60^\circ = \frac{\cancel{60^\circ} \cdot \pi}{\cancel{180^\circ}_{3}} = \underline{\underline{\frac{\pi}{3}}}$$

(9)

Största värde $= 5 + A$

Minsta värde $= 5 - A$

$$5 + A = 2 \cdot (5 - A)$$

$$5 + A = 10 - 2A$$

$$3A = 5$$

$$\underline{\underline{A = \frac{5}{3}}}$$

(10) $f(x) = 35 - 4 \cdot \cos^2 x$

$f'(x) = (-4) \cdot 2 \cdot \cos x \cdot (-\sin x)$

$\underline{\underline{f'(x) = 8 \cdot \cos x \cdot \sin x}}$