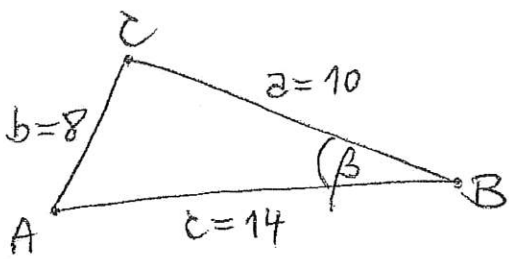


Lösningar till diagnosprov Mat 4 kap 1

(med rättelser och ...)

①



Cosinussatsen:

$$b^2 = a^2 + c^2 - 2ac \cdot \cos \beta$$

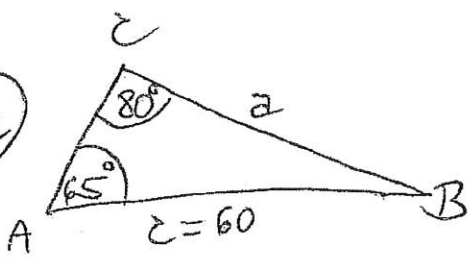
$$2ac \cdot \cos \beta = a^2 + c^2 - b^2$$

$$\cos \beta = \frac{10^2 + 14^2 - 64}{2 \cdot 10 \cdot 14}$$

$$\beta = \cos^{-1}\left(\frac{100 + 196 - 64}{280}\right)$$

$$\underline{\underline{\beta = 34^\circ}}$$

②



Sinussatsen:

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 65^\circ}{a} = \frac{\sin 80^\circ}{60}$$

$$\frac{60 \cdot \sin 65^\circ}{\sin 80^\circ} = a$$

$$B = 180^\circ - (65^\circ + 80^\circ)$$

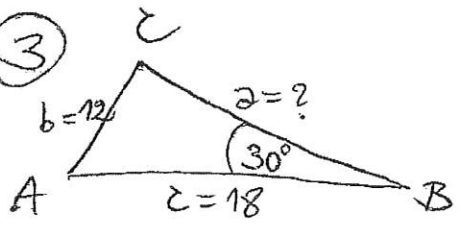
$$B = 35^\circ$$

Areasatsen: $A = \frac{a \cdot c \cdot \sin B}{2} = \frac{60 \cdot \sin 65^\circ \cdot 60 \cdot \sin 35^\circ}{2 \cdot \sin 80^\circ}$

(OBS! i grader)

$$\underline{\underline{A = 950,14}}$$

③



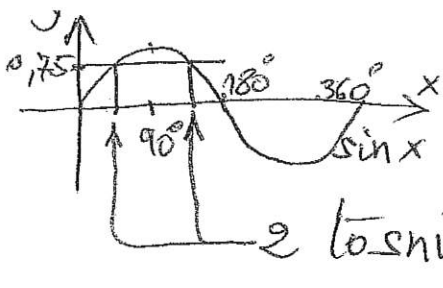
Sinussatsen:

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\sin C = \frac{18 \cdot \sin 30^\circ}{12} = \frac{3}{2}$$

$$\sin C = \frac{3}{4}$$

$$\sin B = \sin 30^\circ = \frac{1}{2}$$



$$C = \sin^{-1}(0,75)$$

$$\begin{cases} C_1 = 48,5904^\circ \\ C_2 = 180^\circ - C_1 = 131,4096^\circ \end{cases}$$

$$A_1 = 180^\circ - 30^\circ - C_1 = 101,4096^\circ$$

$$A_2 = 180^\circ - 30^\circ - C_2 = 18,5904^\circ$$

Sinussatsen: $\frac{\sin A}{a} = \frac{\sin B}{b}$

$$\frac{\sin 101,4096}{a_1} = \frac{1}{24}$$

$$24 \cdot \sin 101,4096 = a_1$$

$$\underline{\underline{a_1 = 23,53}}$$

$$24 \cdot \sin 18,5904 = a_2$$

$$\underline{\underline{a_2 = 7,65}}$$

Se bilaga med ritning.

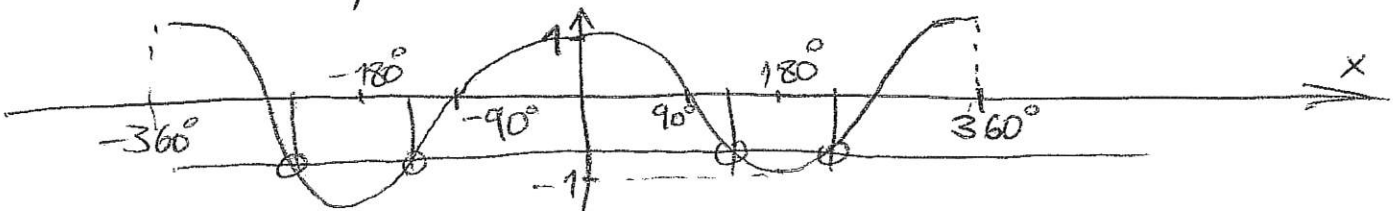
2 lösningar
2 svar
båda o.k.

④ $\cos v = -0,845$

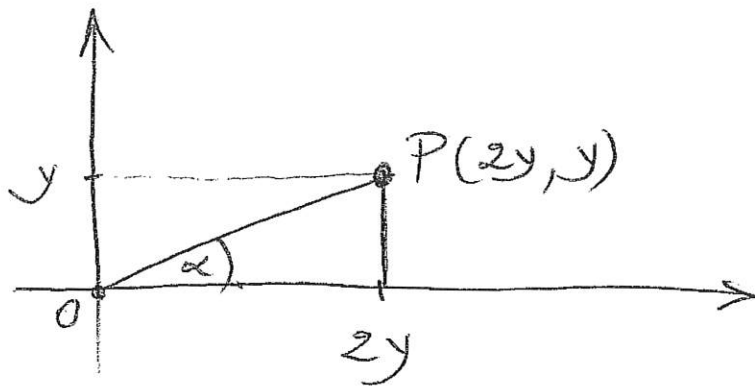
$$v = \cos^{-1}(-0,845) = 147,67^\circ$$

Svar: $\begin{cases} 147,67^\circ + n \cdot 360^\circ \\ -147,67^\circ - n \cdot 360^\circ \end{cases} \quad (n \text{ heltal})$

Räcker för n kan även vara negativt.



5



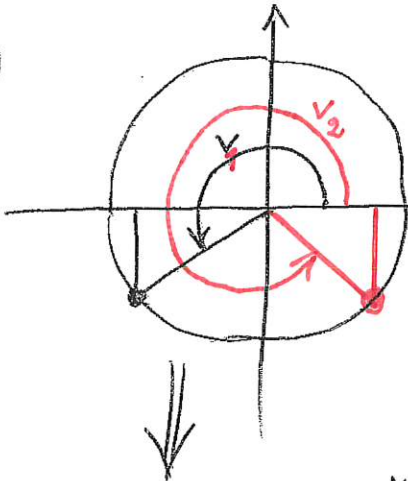
$$\tan \alpha = \frac{y}{2y}$$

$$\tan \alpha = \frac{1}{2}$$

$$\alpha = \tan^{-1}\left(\frac{1}{2}\right)$$

$$\alpha = \underline{\underline{26,6^\circ}}$$

6



$$\sin^2 v + \cos^2 v = 1$$

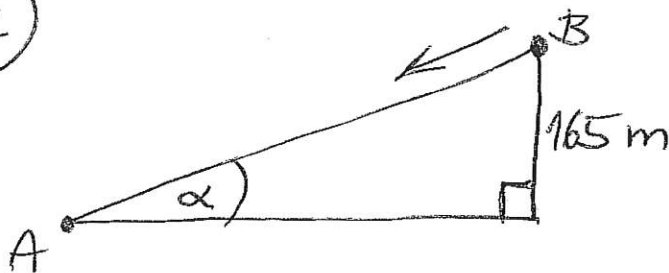
$$\left(-\frac{3}{5}\right)^2 + \cos^2 v = 1$$

$$\cos^2 v = 1 - \frac{9}{25} = \frac{16}{25}$$

$$\cos v = \left(\pm \frac{4}{5}\right) \text{ OBS! Det finns två lösningar.}$$

Svar: $\cos v = \pm \frac{4}{5}$

7



AB = nedfartens längd

$$67 \text{ m/s} = \frac{AB}{10 \text{ s}}$$

$$670 \text{ m} = AB$$

$$\sin \alpha = \frac{165 \text{ m}}{670 \text{ m}} = 0,2463$$

$$\alpha = \sin^{-1}(0,2463)$$

$$\alpha = \underline{\underline{14^\circ}}$$

$$\begin{aligned} \frac{x}{4} &= 180^\circ \\ x &= 4 \cdot 180^\circ \\ x &= 720^\circ \end{aligned}$$

8 $\tan\left(\frac{x}{4}\right) = \frac{1}{2}$

$$\frac{x}{4} = \tan^{-1}\left(\frac{1}{2}\right) = 26,565^\circ$$

$$x = 4 \cdot 26,565^\circ = 106,26^\circ$$

Svar: 720°
 $106^\circ + n \cdot 360^\circ$ (n heltal)

$$(9) \quad 14,4 \cdot \cos x + 6 \cdot \sin x = 0$$

$$6 \cdot \sin x = -14,4 \cdot \cos x \quad | / \cos x$$

$$6 \cdot \frac{\sin x}{\cos x} = -14,4 \cdot \frac{\cos x}{\cos x}$$

$$6 \cdot \tan x = -14,4$$

$$\tan x = -\frac{14,4}{6}$$

$$x = \tan^{-1} \left(-\frac{14,4}{6} \right)$$

$$x = -67,38^\circ$$

Perioden för tangens är 180°
~~180~~

Svar: $x = -67,38^\circ + n \cdot \cancel{360}$

eller \hookrightarrow $x = \cancel{292,6^\circ} + n \cdot \cancel{360}$ ¹⁸⁰

$$(10) \quad \text{Påståendet: } \tan x = \frac{\sin 2x}{1 + \cos 2x}$$

Bevis: HL = $\frac{2 \cdot \sin x \cdot \cos x}{1 + \cos^2 x - \sin^2 x} =$

$$= \frac{2 \cdot \sin x \cdot \cos x}{1 + \cos^2 x - (1 - \cos^2 x)} = \frac{2 \cdot \sin x \cdot \cos x}{1 + \cos^2 x + \cos^2 x} =$$

$$= \frac{\cancel{2} \cdot \sin x \cdot \cancel{\cos x}}{\cancel{2} \cos^2 x} = \frac{\sin x}{\cos x} = \tan x = VL$$

(11) Påstående: $\frac{\cos x}{1 + \sin x} = \frac{1}{\cos x} - \tan x$

Förlängning med $(1 - \sin x)$

Bevis: VL = $\frac{\cos x}{1 + \sin x} = \frac{\cos x \cdot (1 - \sin x)}{(1 + \sin x) \cdot (1 - \sin x)} =$

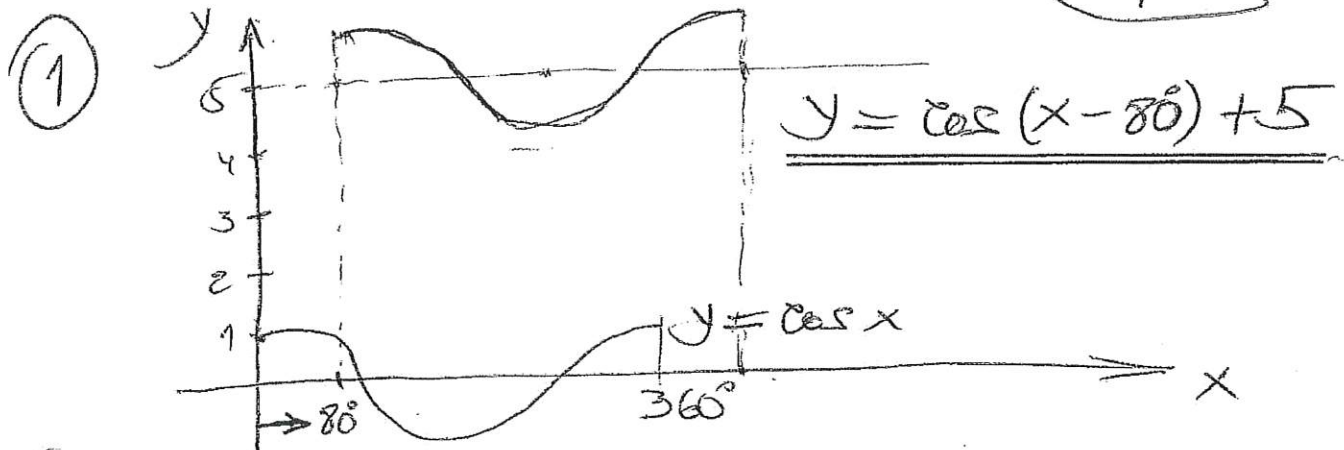
$= \frac{\cos x \cdot (1 - \sin x)}{1 - \sin^2 x} = \frac{\cancel{\cos x} \cdot (1 - \sin x)}{\cos^2 x} =$

$= \frac{1 - \sin x}{\cos x} = \frac{1}{\cos x} - \frac{\sin x}{\cos x} =$

$= \frac{1}{\cos x} - \tan x = HL$

Lösningar till diagnosprov Ma 4

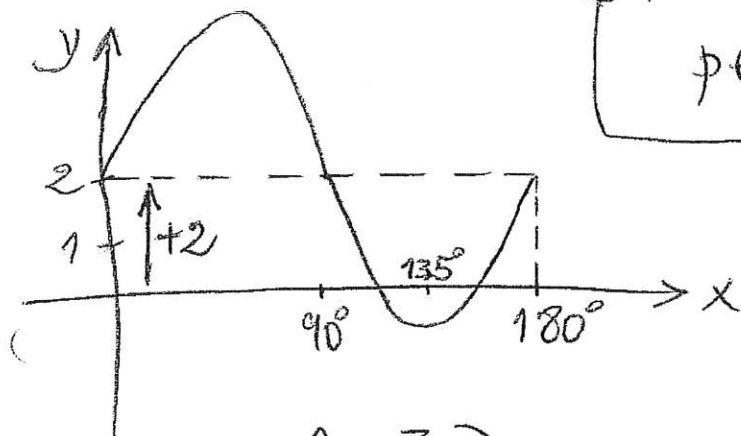
kap 2



② Amplituden $A = \frac{y_{\max} - y_{\min}}{2} = \frac{5 - (-1)}{2} = 3$

Frekvensen $k = 2$, därför att:

perioden $= 180^\circ = \frac{360^\circ}{2}$



Förskjuten i y-led
med 2 enheter

$B = 2$

Svar: $A = 3$
 $B = 2$
 $k = 2$

$y = 3 \cdot \sin 2x + 2$

③ $A = \frac{5 - (-25)}{2} = 15$

$k = \frac{360^\circ}{40^\circ} = 9$ \rightarrow kurvans period

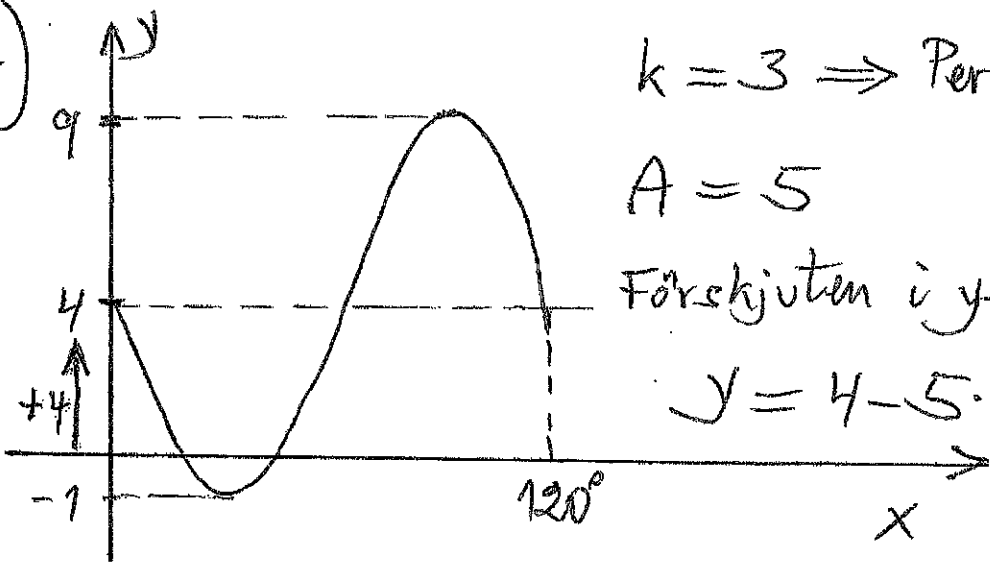
$y = 15 \cdot \cos 9x - 10$

(Beteckningar: se ovan)

$B = -10$: Förskjutning i y-led (nedåt).

Därför att $A = 15$ och grafen visar 5 som max-punkt på y-axeln. \Rightarrow Kurvan har förskjutits från 15 till 5, dvs 10 enheter.

4



$$k = 3 \Rightarrow \text{Period} = \frac{360^\circ}{3} = 120^\circ$$

$$A = 5$$

Förskjutet i y-led med 4 enheter:

$$y = 4 - 5 \cdot \sin 3x$$

Svar:

$$\text{Period} = 120^\circ$$

$$\text{Största värde} = 4 + 5 = 9$$

$$\text{Minsta värde} = 4 - 5 = -1$$

5

$$\text{Period} = \frac{360^\circ}{2} = 180^\circ$$

$$A = 10$$

$$\text{Förskjutning i y-led} = 10$$

$$\text{Förskjutning i x-led} = 10^\circ$$

OBS!

$$\dots \sin(2x - 20^\circ)$$

$$\frac{20^\circ}{2} = 10^\circ$$

$$20^\circ = 10^\circ \cdot 2$$

$$\text{Max-värde} = 10 + 10 = 20$$

$$\text{Min-värde} = 10 - 10 = 0$$

6

$$\text{Period} = 720^\circ \Rightarrow k = \frac{360^\circ}{720^\circ} = \frac{1}{2}$$

$$\text{Förskjutning i x-led} = 120^\circ$$

$$A = 1$$

Ingen förskjutning i y-led:

$$\underline{\underline{y = \sin\left(\frac{x}{2} - 60^\circ\right)}}$$

2x
samma
anledning

$$120^\circ \cdot \frac{1}{2} = 60^\circ$$

Pröva även:
 $\sin\left(\frac{120^\circ}{2} - 60^\circ\right) =$
 $\sin 0^\circ = 0$

7

$$1 \text{ rad} = \frac{180^\circ}{\pi}$$

$$\frac{5\pi}{36} \text{ rad} = \frac{5\pi}{36} \cdot \frac{180^\circ}{\pi} = \frac{\overset{\text{Förkortat: } 10^\circ}{5\cancel{\pi}} \cdot \cancel{180^\circ}}{\cancel{36} \cdot \cancel{\pi}} = \underline{\underline{25^\circ}}$$

8

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

$$60^\circ = \frac{\cancel{60^\circ} \cdot \pi}{\underset{3}{180^\circ}} = \underline{\underline{\frac{\pi}{3}}}$$

Förkortat

9

$$\text{Största värde} = 5 + A$$

$$\text{Minsta värde} = 5 - A$$

$$5 + A = 2 \cdot (5 - A)$$

$$5 + A = 10 - 2A$$

$$3A = 5$$

$$\underline{\underline{A = \frac{5}{3}}}$$

10

$$f(x) = 35 - 4 \cdot \cos^2 x$$

$$f'(x) = (-4) \cdot 2 \cdot \cos x \cdot (-\sin x)$$

$$\underline{\underline{f'(x) = 8 \cdot \cos x \cdot \sin x}}$$

3

Bilaga

Givet: Sidorna $b = 12$ och $c = 18$

Vinkel vid B : 30°

Sökt: Sidan a

Det finns två

lösningar:

(Se uträkning i facit.)

